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Microscopic insight into nuclear structure properties of Dysprosium nuclei

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Abstract: Various nuclear structure quantities for $^{154-166}$Dy nuclei have been calculated using Variation-After-Projection (VAP) framework in conjunction with Hartree-Bogoliubov (HB) ansatz. The yrast spectra with $J^{\pi} \leq 16^{+}$, $B(E2)$ transition probabilities and occupation numbers for various shell model orbits have been obtained. The observed onset of deformation in going from ^{154}Dy to ^{166}Dy has been explained in terms of the enhanced occupation of $(h_{11/2})_{\pi}$, $(i_{13/2})_{\upsilon}$ and $(h_{9/2})_{\upsilon}$ orbits and increased polarization of $(d_{5/2})_{\pi}$ orbit .

Key words: Hartree-Bogoliubov calculations; nuclear structure properties of $^{154-166}$Dy; $B(E2)$ transition probabilities; sub-shell occupation probabilities.

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1. Introduction

In the last few years much progress has been made in the understanding of nuclear structure problems in the spectroscopy of heavy nuclei. Most of these studies have been devoted to the yrast region where many new phenomenon like, damping of the rotational motion, pairing phase transitions and shape changes have been discovered through the study of discrete lines. In recent years, many phenomenological observations regarding the nuclei in the spherical to deformed shape transitional region have led to renewed interest in the rare-earth region. The Dysprosium (Dy) isotopes, which are members of the well known prolate deformed nuclei, are the main examples under such category. Some nuclei in this region, for example, $^{152}_{66}Dy_{86}$ [1-3] are found to be near-spherical oblate in shape at low spin and exhibit coexisting collective prolate shapes at higher spins. Other nuclei, such as $^{154}_{66}Dy_{88}$ [4] are collective prolate rotors at low spin and become non collective oblate at high spin. Nuclei with higher N (≥90), such as $^{156}_{66}Dy_{90}$ [5], can often be characterized as collective prolate rotors throughout their entire spin range.

Much information on the structure of Dysprosium nuclei around the mass number A≈150 can also be extracted from the different experiments applied to these nuclei using different experimental techniques. For example, Caprio et al [6] studied the level scheme and structure of ^{156}Dy nucleus through γ-ray spectroscopy. From the nuclear reaction spectroscopy [7-11] and Coulomb excitation measurements [12,13], extensive information on the nuclear structure of ^{154,156}Dy and ^{162}Dy has been obtained.

The most interesting feature of the observed energy spectra (see Table 1) is the variation of E_2^+ excitation energy from ^{154}Dy to ^{166}Dy. Whereas the value of E_2^+_excitation energy in case of ^{154}Dy is 0.3MeV, it sharply drops to 0.1MeV for ^{156}Dy and thereafter it decreases slowly to 0.07MeV till ^{166}Dy is reached. Besides this, the E_4^+/E_2^+ value increases suddenly from 2.33 for ^{154}Dy to 2.93 for ^{156}Dy and after that, there is a slow increase in the value of E_4^+/E_2^+ ratio from $^{158-166}$Dy. It has been pointed out by Zhang *et al* [14] that the ratio E_4^+/E_2^+ is an important parameter for determining the shape of a nucleus. For a rigid rotator, its value should be 3.33 whereas its value for a spherical nucleus should be around 2. The values in between these two limits indicate that the nucleus is quasi-deformed and has vibrational character. Thus from the observed spectra in neutron rich $^{154-166}$Dy isotopes, it appears that ^{154}Dy isotope is quasi-deformed and there is a sudden increase in the degree of deformation as one moves from ^{154}Dy to ^{156}Dy. After that, there is a slow increase in the degree of deformation in moving from $^{158-166}$Dy isotopes. The observed trend is also confirmed by the systematics of $B(E2;\ 0^+ \rightarrow 2^+)$ values given in Table 4. Whereas for ^{154}Dy, the $B(E2;\ 0^+ \rightarrow 2^+)$ value is of the order of 2.39 ± 0.13 $e^2b_n^2$, it increases suddenly to 3.71 ± 0.40 $e^2b_n^2$ for ^{156}Dy and after that it increases slowly to 5.60 ± 0.05 $e^2b_n^2$ till ^{164}Dy is reached. It seems that at least for the isotopes $^{154-164}$Dy, there is some correlation between $B(E2;\ 0^+ \rightarrow 2^+)$ and the excitation energy of the 2_1^+ state (ΔE). As ΔE decreases, $B(E2;\ 0^+ \rightarrow 2^+)$ increases in its absolute value. This type of inverse correlation between $B(E2;\ 0^+ \rightarrow 2^+)$ values and ΔE is in fact consistent with the Grodzins rule [15]. Grodzins has indeed developed a quantitative formula relating the two quantities. Raman *et al* [16] recently updated Grodzins work based on all current data.

In contrast to the large-scale effort that has been made on the experimental side, only a few theoretical

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models [17-21] have been proposed to explain the structure of these nuclei. The complex nuclear structure of softly deformed shape transitional nuclei at N=88-90 in A=150-200 region has been a challenge to the collective nuclear structure theory. Here, the features of both a spherical vibrator and a deformed rotor are present in the same nucleus. The level structures, absolute E2 moments, relative *B (E2)* values for interband transitions and grouping into quasibands of ^{154}Dy and ^{156}Dy isotopes were derived using the dynamic pairing-plus-quadrupole model [17] and other models also like boson expansion model, rotation-vibration model (RVM), interacting boson model (IBM-I) given in the references therein. This analysis was extended to five quasibands in ^{154}Dy and seven collective *K* bands in ^{156}Dy. Sometime back, V.K.B. Kota [18] studied the low-lying spectra of heavy deformed even-even nuclei like ^{164}Dy, ^{166}Er and ^{168}Yb and also calculated various interband and B(E2) values quite extensively in the framework of Pseudo SU(3) model. The energy levels of ground state bands, β bands and γ bands of $^{154-162}$Dy isotopes were also studied by Chuu *et al* [19] in the model of traditional interacting boson approximation (IBA). They also calculated the Yrast *B (E2)* values for these isotopes. The Yrast lines of even-even Dy isotopes have been studied in detail in the framework of the projected shell model (PSM) [20,21].

From the thorough investigation of the research work done on Dy nuclei in the perspective theoretical frameworks, it has been found that various models which have been applied to study the structure of even-even Dy isotopes so far, are found to cover only a few isotopes in the $^{154-166}$Dy isotopic mass chain and hence the clarity regarding the overall deformation systematics of Dy isotopes is not there. Thus, for a complete analysis, there is a need of a more elaborate microscopic nuclear theory. In view of this, we have planned to study the systematics of neutron rich even-even Dy isotopes, in a suitable microscopic calculational framework. As it was shown by Khosa and Sharma [22], two body effective interactions have a dominantly quadrupole-quadrupole character and the deformation producing tendency of neutron-proton (np) and like particle interactions depends upon the degeneracy of the underlying single particle valence space. One of the natural choices for the two-body residual interaction would, therefore, be pairing plus quadrupole-quadrupole (PQ) interaction. We have, however, found it important to include a correction energy term to the PQ interaction in the form of hexadecapole-hexadecapole interaction, which hereafter will be denoted as (PQH) interaction. We have thus, calculated various nuclear structure quantities like Yrast spectra, $B(E2)$ transition probabilities and sub-shell occupation numbers for even-even $^{154-166}$Dy-isotopes by employing PQH interaction, operating in a valence space spanned by $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $2f_{7/2}$, $1g_{7/2}$, $1h_{9/2}$, $1h_{11/2}$ and $1i_{13/2}$ orbits for protons and neutrons. The doubly closed nucleus ^{100}Sn has been taken as inert core.

A comparison of the calculated Yrast spectra obtained with PQH interaction with the observed Yrast states indicates that the agreement is satisfactory, thereby indicating that inclusion of hexadecapole interaction plays an important part in obtaining satisfactory agreement with the experiment.

2. Materials and Methods

2.1 The one and two-body parts of Hamiltonian

In our calculations, we have employed the valence space spanned by $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $2f_{7/2}$, $1g_{7/2}$, $1h_{9/2}$, $1h_{11/2}$ and $1i_{13/2}$ orbits for protons and neutrons under the assumption of N=Z=50 sub-shell closure. The single particle energies (S.P.E's) that we have taken are (in MeV):

$(3s_{1/2})$ = 1.4, $(2d_{3/2})$ = 2.0, $(2d_{5/2})$ = 0.0, $(2f_{7/2})$ = 10.9, $(1g_{7/2})$ = 2.5, $(1h_{9/2})$ = 11.5, $(1h_{11/2})$ = 4.0, and $(1i_{13/2})$ = 13.5.

The S.P.E's of $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, and $1h_{11/2}$ are exactly the same as those employed by Vergados and Kuo [23] as well as Federman and Pittel [24]. The S.P.E's of $2f_{7/2}$, $1h_{9/2}$, and $1i_{13/2}$ orbits are taken from Nilsson diagrams, published in the book [25]. The two-body effective interaction that has been employed is of pairing-plus-quadrupole-quadrupole-plus-hexadecapole-hexadecapole (PQH) type. The strengths for the like particle neutron-neutron (n-n), proton-proton (p-p) and neutron-proton (n-p) components of the quadrupole-quadrupole (q.q) interaction were taken as:

$\chi_{nn}(=\chi_{pp})$ = -0.0069 MeV b^{-4}, and

χ_{np} = -0.0142 MeV b^{-4} .

Here, $b = \sqrt{\hbar / m\omega}$ is the oscillator parameter. The strength for the pairing interaction was fixed through the approximate relation $G = (18–21) / A$.

The relative magnitudes of the parameters of the hexadecapole-hexadecapole parts of the two-body interaction were calculated from a relation suggested by Bohr and Mottelson [26]. According to them, the approximate magnitude of these coupling constants for isospin T=0 is given by:

$$\chi_{\lambda} = \frac{4\pi}{2\lambda+1}\frac{m\omega_0^2}{A<r^{2\lambda-2}>} \text{ for } \lambda = 1,2,3,4 \qquad \ldots\ldots (1)$$

and the parameters for T = 1 case are approximately half the magnitude of their T = 0 counterparts. This relation was used to calculate the values of χ_{pp4} relative to χ_{pp} by generating the wave function for dysprosium-isotopes and then calculating the values of $< r^{2\lambda-2} >$ for λ= 2 and 4.

The values for hexadecapole-hexadecapole part of the two-body interaction turn out to be

$\chi_{pp4}(=\chi_{nn4})$ = -0.00010 MeV b^{-8}, and
χ_{pn4} = -0.00020 MeV b^{-8}.

2.2 Projection of states of good angular momentum from axially - symmetric HB intrinsic states

The procedure for obtaining the axially symmetric HB intrinsic states has been discussed in Ref. [27].

3. Results and Discussion

3.1 Deformation systematics of Dy-isotopes

We first discuss here the systematics of E_2^+ in $^{154-166}$Dy. In Table 1, the experimental values of excitation energy of E_2^+ state (*ΔE*), intrinsic quadrupole moments of HB states, the ratio of intrinsic quadrupole moment to maximum possible value of quadrupole moment ($<Q_0^2>_{HB}$ / $<Q_0^2>_{max.}$= RQ) obtained with PQH interaction and the experimental values of E_4^+/E_2^+ ratio for $^{154-166}$Dy isotopes are presented. From the systematics of E_2^+ excitation energy in $^{154-166}$Dy (see Table 1), it is observed that the energy of 2_1^+ (*ΔE*) states decreases suddenly from its value of 0.3MeV in ^{154}Dy to 0.1MeV in ^{156}Dy and thereafter, the value of (*ΔE*) decreases slowly to 0.07MeV till ^{166}Dy is reached. This indicates that there is an increase in the degree of deformation as we move from ^{154}Dy to ^{166}Dy. Phenomenologically, it is well known from Grodzins rule [15] that a nucleus having a smaller energy gap *ΔE* should have a larger quadrupole moment for 2^+ state. Since quadrupole moment of second excited state (Q_2^+) of a nucleus is directly related to its intrinsic quadrupole moment, one should, therefore, expect that a smaller energy gap *ΔE* should manifest itself in terms of a larger value for the ratio RQ of intrinsic quadrupole moment to maximum possible value of quadrupole moment for that nucleus in the SU(3) limit and *vice versa*. In other words, the observed systematics of E_2^+ with '*A*' should produce a corresponding inverse systematics of this ratio of quadrupole moments for the $^{154-166}$Dy nuclei with increasing '*A*'. Based on the above logic, the calculated values of this ratio should, therefore, exhibit an increasing trend as we move from ^{154}Dy to ^{166}Dy. In Table 1, the results of HB calculations are presented.

Table 1. The experimental values of excitation energy of the E_2^+ state (*ΔE*) in MeV, intrinsic quadrupole moments of proton ($<Q_0^2>_\pi$) , neutron($<Q_0^2>_\upsilon$) and the HB states $<Q_0^2>_{HB}$, maximum possible value of quadrupole moment $<Q_0^2>_{max.}$ obtained with PQH interaction, the ratio of intrinsic quadrupole moment to maximum possible value of quadrupole moment ($<Q_0^2>_{HB}$ / $<Q_0^2>_{max.}$ = RQ) and the experimental values of E_4^+/E_2^+ ratio for $^{154-166}$Dy isotopes. The quadrupole moments have been computed in units of b^2, where $b = \sqrt{\hbar / m\omega}$ is the oscillator parameter.

Dy- nuclei (A)	PQH Interaction						
	E_2^+ (Expt.)*	$<Q_0^2>_\pi$	$<Q_0^2>_\upsilon$	$<Q_0^2>_{HB}$	$<Q_0^2>_{max}$	RQ	E_4^+/E_2^+ (Expt.)*
154	0.30	41.15	22.8	63.95	167.14	0.38	2.23
156	0.10	43.23	26.77	70	165.08	0.42	2.93
158	0.09	47.82	33.43	81.25	161.40	0.50	3.20
160	0.08	50.99	37.74	88.73	156.36	0.56	3.27
162	0.08	52.20	40.37	92.57	151.70	0.61	3.29
164	0.07	52.89	44.64	97.53	145.35	0.67	3.30
166	0.07	53.30	43.14	96.44	138.02	0.69	3.31
158	0.09	47.82	33.43	81.25	161.40	0.50	3.20
160	0.08	50.99	37.74	88.73	156.36	0.56	3.27

*Data taken from Ref. [33]

As is evident from the results presented in Table 1, the deformation appearing in the heavy Dy isotopes is more than 65% of the maximum possible deformation in these isotopes. It is indicated by the fact that the ratio RQ increases from its value of 0.38 to 0.69 as we move from ^{154}Dy to ^{166}Dy.This fact is also confirmed by the change in the value of the ratio E_4^+/E_2^+. The value of this ratio increases suddenly from 2.33 for ^{154}Dy to 2.93 for ^{156}Dy and thereafter, this value increases slowly till ^{166}Dy is reached. Thus, we can say that the results on the ratio of quadrupole moments are consistent with E_2^+ systematics.

We now discuss and highlight some of the well-accepted factors responsible for bringing in sizeable collectivity in these Dy nuclei in general. It is well known that if the down slopping components of a high-*j* valence orbit starts filling up, it has the effect of bringing in sharp increase in collectivity. Besides this, it is also known that a closed shell or a sub-shell makes zero contribution to the intrinsic quadrupole moment. Therefore, if a sub-shell gets polarized and still has occupation probability greater than the mid sub- shell occupation, then it will again have the effect of introducing some degree of deformation in the nucleus. In addition, the role of n-p interaction in the SOP (spin-orbit partner) orbits in the context of the general development of collective features was also suggested by Federman and coworkers [28, 29] and by Casten *et al* [30]. Their calculation provided evidence suggesting the neutron-proton interaction between the valence nucleons in the SOP orbits- the orbits $(g_{9/2})_\pi$ and $(g_{7/2})_\upsilon$ in the Zr and Mo region- may be instrumental *vis-à-vis*, the observed onset of deformation in the Mo-isotopes with A>100. The subscript π stands for proton and υ stands for neutron. In the light of above effects, it is now tried to find out the causes responsible for the observed systematics of $^{154-166}$Dy-isotopes. In order to understand how deformation arises in these isotopes, we present in Tables 2 and 3, the results of occupation probabilities of various proton and neutron sub-shells for the ground state.

Table 2. The sub - shell occupation numbers (protons) in the nuclei $^{154-166}$Dy obtained with PQH interaction

Dy nuclei	Sub-shell occupation number							
(A)	$3s_{1/2}$	$2d_{3/2}$	$2d_{5/2}$	$2f_{7/2}$	$1g_{7/2}$	$1h_{9/2}$	$1h_{11/2}$	$1i_{13/2}$
154	1.12	1.67	5.55	0.25	3.57	0.00	3.67	0.13
156	1.06	1.66	5.43	0.29	3.62	0.01	3.75	0.13
158	0.93	1.65	4.89	0.44	3.75	0.01	4.18	0.11
160	0.88	1.65	4.45	0.53	3.72	0.02	4.64	0.07
162	0.86	1.66	4.24	0.57	3.66	0.03	4.78	0.16
164	0.86	1.66	4.15	0.59	3.63	0.03	4.88	0.17
166	0.89	1.71	4.11	0.60	3.70	0.02	4.95	0.00

Table 3. The sub - shell occupation numbers (neutrons) in the nuclei $^{154-166}$Dy obtained with PQH interaction

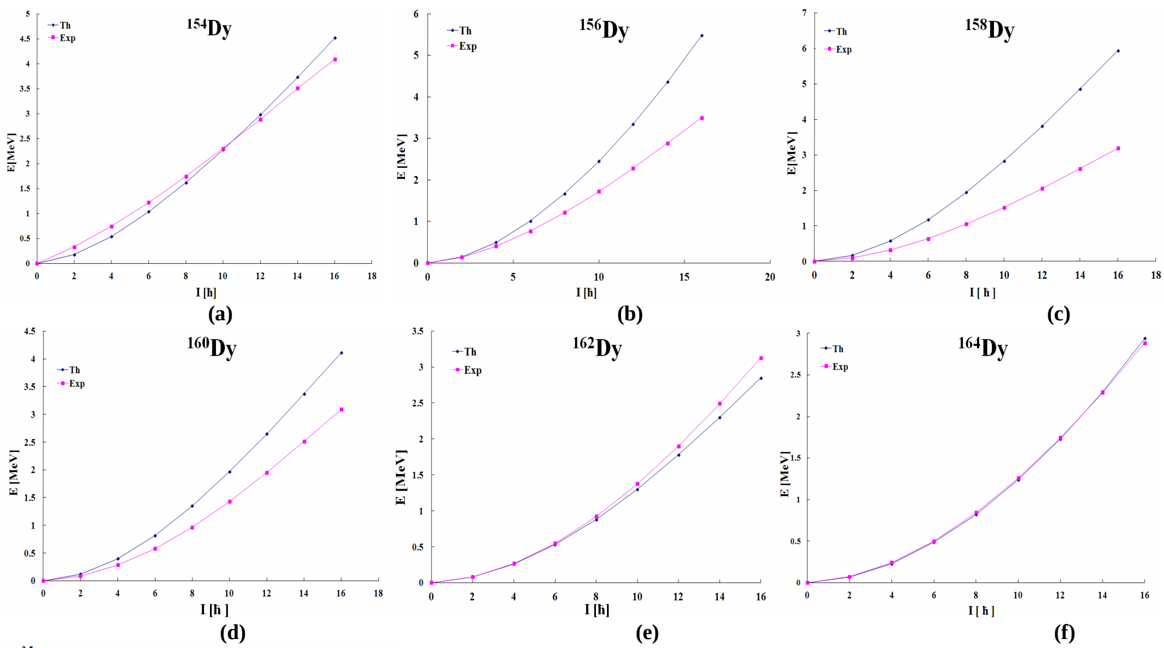
Dy nuclei	Sub-shell occupation number							
(A)	$3s_{1/2}$	$2d_{3/2}$	$2d_{5/2}$	$2f_{7/2}$	$1g_{7/2}$	$1h_{9/2}$	$1h_{11/2}$	$1i_{13/2}$
154	1.99	3.99	5.99	2.51	7.99	3.38	11.82	0.28
156	1.99	3.99	5.99	3.22	7.99	3.89	11.75	1.13
158	1.99	3.99	5.99	3.41	7.98	4.20	11.68	2.70
160	1.99	3.99	5.99	3.82	7.99	4.63	11.71	3.84
162	1.99	3.99	5.99	4.23	7.99	5.50	11.79	4.47
164	1.99	3.99	5.99	4.34	7.99	5.85	11.80	6.00
166	1.99	3.99	5.99	5.39	7.99	6.04	11.80	6.76

These results have been obtained by using the pairing-plus-quadrupole-quadrupole-plus-hexadecapole-hexadecapole (PQH) interaction. It may be noted from these Tables that, for ^{154}Dy, the neutron sub-shells $s_{1/2}$, $d_{3/2}$, $d_{5/2}$ and $g_{7/2}$ are nearly full and $h_{11/2}$ neutron orbit has 11.82 neutrons. Besides this, the $d_{5/2}$ proton orbit is more than two-third full and $h_{9/2}$ proton orbit has zero occupation. Because of the closure of the most of the neutron sub-shells in the valence space and the $(d_{5/2})_\pi$ sub-shell being more than two-third full, the RQ value for ^{154}Dy is less than that of the other Dy isotopes and, therefore, is less deformed. As we move away from ^{154}Dy, it is observed that the $(d_{5/2})_\pi$ orbit gets more polarized due to decrease in its occupation. For example, the occupation of $(d_{5/2})_\pi$ orbit for ^{154}Dy is 5.55 units whereas for ^{166}Dy, it is 4.11 units. Besides this, the $(h_{11/2})_\pi$ occupation goes on increasing in its value for ^{154}Dy onwards towards the higher mass side. Further, from Table 3, it is also noted that $(f_{7/2})_\upsilon$ and $(h_{9/2})_\upsilon$ occupations increase in their values as one moves from ^{154}Dy to ^{166}Dy. In addition, one also observes a sharp increase in the occupation of down slopping '*k*' components of $(i_{13/2})_\upsilon$ orbits. This systematic increase in the occupation of $(h_{11/2})_\pi$, $(f_{7/2})_\upsilon$, $(h_{9/2})_\upsilon$ and $(i_{13/2})_\upsilon$ orbits is responsible for the increase of RQ value as one moves from ^{154}Dy towards higher mass side. From what has been said, the overall observed deformation systematics in $^{154-166}$Dy could be understood in terms of the systematic changes in occupation probabilities of the various valence orbits as explained above.

We now explain the observed systematics of deformation in Dy isotopes with A≥154 in the light of np-interaction between SOP (spin orbit partner) orbits. Note that $(h_{11/2})_\pi$ occupation increases from a value of 3.67 for ^{154}Dy to 4.95 for ^{166}Dy and there is also a polarization of $(d_{5/2})_\pi$ orbit. In addition, one also observes a sharp increase in the occupation of down slopping '*k*' components of $(i_{13/2})_\upsilon$ and $(h_{9/2})_\upsilon$ orbits. Thus, there is an increased opportunity for the np-interaction between the SOP-orbits, the $(h_{11/2})_\pi$ and $(h_{9/2})_\upsilon$ orbits, to operate effectively. All these factors reinforce each other and bring in large collectivity for Dy isotopes. These factors are, therefore, responsible for the observed deformation systematics in Dy isotopes with A>154.

3.2 Yrast spectra

In order to test the reliability of HB calculations, it is important to obtain satisfactory agreement for the Yrast spectra. A calculation for the energy spectra of $^{154-166}$Dy was carried out by employing the phenomenological PQH model of two-body interaction. In Figures 1(a) – 1(g), the Yrast spectra for $^{154-166}$Dy are displayed, where we have compared the experimental values of Yrast states with the theoretical values. The spectra corresponding to Th. are obtained when PQH model of interaction is employed. It has been observed from our calculations that calculated spectra corresponding to Th. reproduce the observed Yrast spectra for $^{154-166}$Dy with reasonably acceptable discrepancy. This level of agreement can be considered to be satisfactory because of a number of considerations. First, the calculation of Yrast spectra is a complex, many body calculations involving a minimum of 54 valence particles for ^{154}Dy and a maximum of 66 valence particles for ^{166}Dy. Another noticeable fact is that the calculations are carried out for the entire set of $^{154-166}$Dy isotopes with a single set of input parameters.



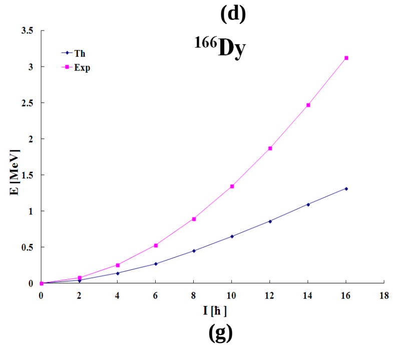


Fig. 1(a-g): (a) Comparison of the observed (Expt.) and theoretical (Th.) low-lying Yrast spectra in ^{154}Dy isotope, Expt. data taken from Ref. [33], **(b)** Comparison of the observed (Expt.) and theoretical (Th.) low-lying Yrast spectra in ^{156}Dy isotope, Expt. data taken from Ref. [34], **(c)** Comparison of the observed (Expt.) and theoretical (Th.) low-lying Yrast spectra in ^{158}Dy isotope, Expt. data taken from Ref. [35], **(d)** Comparison of the observed (Expt.) and theoretical (Th.) low-lying Yrast spectra in ^{160}Dy isotope, Expt. data taken from Ref. [36], **(e)** Comparison of the observed (Expt.) and theoretical (Th.) low-lying Yrast spectra in ^{162}Dy isotope, Expt. data taken from Ref. [37], **(f)** Comparison of the observed (Expt.) and theoretical (Th.) low-lying Yrast spectra in ^{164}Dy isotope, Expt. data taken from Ref. [38] and **(g)** Comparison of the observed (Expt.) and theoretical (Th.) low-lying Yrast spectra in ^{166}Dy isotope, Expt. data taken from Ref. [39].

3.3 Transition probabilities

We next move to the calculation of the reduced E2 transition probabilities i.e., $B(E2;0^+\rightarrow 2^+)$ values. We have calculated the $B(E2)$ transition probabilities for $^{154-166}$Dy isotopes from the values of intrinsic quadrupole moments of protons and neutrons obtained with PQH interaction, by using the rotational model formula given by Ripka and Adler *et al* [31]. According to this formula, if the expectation value of $\hat{J}^2$ is large, the intrinsic electric quadrupole moment is related to the B(E2; $J_i^+\rightarrow J_f^+$) for E2 transitions between the states projected from the intrinsic HFB state, by:

$$B(E2;J_i^+\rightarrow J_f^+) = 5/16\pi \begin{bmatrix} J_i & 2 & J_f \\ 0 & 0 & 0 \end{bmatrix}^2 [e_\pi <Q_0^2>_\pi + e_\upsilon <Q_0^2>_\upsilon]^2 \qquad \ldots\ldots (2)$$

where $<Q_0^2>_\pi$ ($<Q_0^2>_\upsilon$) are the intrinsic quadrupole moments of valence protons (neutrons) and the proton (neutron) effective charges $e_\pi(e_\upsilon)$ are

$e_\pi = (1+e_{eff.})$ and $e_\upsilon = (e_{eff.})$.

In Table 4, we present a comparison of the calculated $B(E2)$ values obtained with PQH interaction with the experimental values [32] for the $0_1^+\rightarrow 2_1^+$ transitions in case of $^{154-166}$Dy. It is noted that the observed $B(E2)$ data for $0_1^+\rightarrow 2_1^+$ transitions shows an increasing trend *e.g.* the observed $B(E2; 0^+\rightarrow 2^+)$ value for ^{154}Dy is 2.39 ± 0.13 $e^2b_n^2$ whereas its value for ^{164}Dy is 5.60 ± 0.05 $e^2b_n^2$. It has been noted that the calculated $B(E2)$ estimates are in satisfactory agreement with the experiments provided one chooses $e_{eff.}$= 0.8 and 0.85. For example, the calculated $B(E2)$ value for ^{154}Dy is 2.44 $e^2b_n^2$ for $e_{eff.}$= 0.8 and for ^{164}Dy, it is 5.65 $e^2b_n^2$ for $e_{eff.}$ = 0.85.

Table 4. Comparison of the calculated and experimental *B*(*E2*; $0_1^+ \rightarrow 2_1^+$) values of $^{154-166}$Dy-isotopes. The *B(E2)* values are in units of $e^2 b_n^2$ where b_n stands for barn (1 barn=10^{-24} cm^2).

Dy Nuclei (A)	*B*(*E2*; $0^+ \rightarrow 2^+$)		
	$e_{eff.}$= 0.8	$e_{eff.}$= 0.85	(Expt.)*
154	2.44	2.61	2.39 ± 0.13
156	2.92	3.13	3.71 ± 0.40
158	3.78	4.06	4.66 ± 0.05
160	4.5	4.84	5.13 ± 0.11
162	4.72	5.24	5.35 ± 0.11
164	5.25	5.65	5.60 ± 0.05
166	5.22	5.61	-

* Data taken from Ref. [32]

The use of effective charges [23] is generally invoked in nuclear structure calculations to represent the contribution made by the core towards the electromagnetic properties, due to its getting polarized as the nucleons are put in the valence space. The valence particles through two-body effective interactions can interact with the core and cause excitations. The degree of polarization of the core is, therefore, expected to increase with increase in the number of valence particles. Since the calculations for the *B(E2)* values depend on the RQ values, so *B(E2)* values should follow the same trend as that followed by the RQ values for $^{154-166}$Dy isotopes. This feature of dysprosium (Dy) isotopes has been reproduced by the present calculations.

4. Conclusions

Based on the results of present calculations, the following broad conclusions can be drawn:

(i) The quasi-deformed character of ^{154}Dy is due to the occupation of k=1/2 components of $h_{11/2}$ and $g_{7/2}$ proton orbits. The fact that it is the least deformed among the Dy isotopes presented here, is due to the sub-shell closure of $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$ and $h_{11/2}$ neutron orbits. Besides this, $d_{5/2}$ proton orbit is more than two-third full.

(ii) The observed increasing trend of deformation as one moves away from ^{154}Dy, is seen to be dependent, sensitively, on the increase in the occupation of $(h_{11/2})_\pi$ orbit and the polarization of $(d_{5/2})_\pi$ orbit. Besides this, there is an increase in the occupation of $(f_{7/2})_\upsilon$, $(h_{9/2})_\upsilon$ and $(i_{13/2})_\upsilon$ orbits. These changes in the occupation of $(d_{5/2})_\pi$, $(h_{11/2})_\pi$ and $(f_{7/2})_\upsilon$, $(h_{9/2})_\upsilon$, $(i_{13/2})_\upsilon$ orbits are responsible for the increase in the value of RQ as one moves from ^{154}Dy to ^{166}Dy.

(iii) The observed increase in the degree of deformation in A≥154 mass chain is seen to arise due to the enhanced occupation of $(h_{11/2})_\pi$ orbit, increased polarization of $(d_{5/2})_\pi$ orbit and increase in the occupation of down slopping '*k*' components of $(i_{13/2})_\upsilon$ and $(h_{9/2})_\upsilon$ orbits. In addition, it is also found that there is an increased opportunity for the n-p interaction between the SOP orbits- the $(h_{11/2})_\pi$ and $(h_{9/2})_\upsilon$ orbits, to operate effectively.

(iv) The values of *B (E2)* transition probabilities calculated with PQH interaction for $^{154-164}$Dy, are found to be in satisfactory agreement with the experiments.

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